HALE SCHOOL

Semester One Examination, 2018

Question/Answer booklet

Year 11
MATHEMATICS METHODS
UNIT 1
Section One:
Calculator Free

Booklet 1 of 3

Student name Marking Key

Circle your teacher's Initials: **IFB** **DD** **VMU** **SWA** **MS** **AGC**

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	66	35
Section Two: Calculator-assumed	13	13	100	78	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section One: Calculator-free **35% (66 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

(a) Solve $5(2t+1)-3(t-4)=0$ for t. (2 marks)

$10t+5-3t+12=0$ ✓ correct expansion of brackets

$\therefore 7t+17=0$

$t=-\frac{17}{7}$ ✓ correct solution

2

(b) Solve $\frac{7}{a-5}-\frac{3}{4a}=0$ for a. (3 marks)

$\frac{7}{a-5}-\frac{3}{4a}=0$

$\therefore \frac{7}{a-5}=\frac{3}{4a}$ ✓ cross multiply terms

$\therefore 28a=3(a-5)$

$\therefore 28a=3a-15$) solves resulting equation

$\therefore 25a=-15$

$a=-\frac{3}{5}$ ✓ correct solution

3

Question 2 **(11 marks)**

Solve the following equations.

(a) $6x^2 = 3x$. (2 marks)

$6x^2 - 3x = 0$

$3x(2x-1) = 0$ ✓ Factorises the equation.

$\therefore 3x = 0, \quad 2x - 1 = 0$

$x = 0, \quad x = \frac{1}{2}$ ✓ solves for both values of x.

(b) $x(x+2) = 24$. (2 marks)

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$ ✓ factorises Trinomial

$x = -6, \quad x = 4$ ✓ solves for both values of x.

Question 2 continued

(c) $x^2 = 4x + 16$ (Express your answer in the simplest form.) (3 marks)

4

$x^2 - 4x - 16 = 0$

$\therefore (x-2)^2 - 4 - 16 = 0$ ✓ completes the square

$(x-2)^2 = 20$

$x - 2 = \pm\sqrt{20}$ ✓ $\sqrt{}$ both sides

$x = 2 \pm \sqrt{20}$ ✓ unsimplified answer

$= 2 \pm 2\sqrt{5}$ ✓ simplified answer

4

(d) $2^{2x+1} - 9\left(2^x\right) + 2^2 = 0$ (4 marks)

$2(2^x)^2 - 9(2^x) + 4 = 0$ ✓ rearranges to a quadratic form

Let $y = 2^x$

$\therefore 2y^2 - 9y + 4 = 0$ ✓ substitutes to get a trinomial

$\therefore (2y - 1)(y - 4) = 0$

$\therefore y = \frac{1}{2}, y = 4$ correct factorisation/solution for y.

$\therefore 2^x = \frac{1}{2}, 2^x = 4$

$\therefore x = -1, x = 2$ ✓ correct solution for x

4

See next page

Question 3 **(10 marks)**

(a) Simplify $\left(\frac{3xy^2}{w}\right)^2 \div \sqrt{\frac{9x^3y^4}{w^6}}$ (4 marks)

$= \frac{9x^2y^4}{w^2} \div \left(\frac{9x^3y^4}{w^6}\right)^{\frac{1}{2}}$ ✓ converts $\sqrt{\ }$ to a power of a $\frac{1}{2}$ correctly.

$= \frac{9x^2y^4}{w^2} \div \frac{3x^{\frac{3}{2}}y^2}{w^3}$

$= \frac{3\cancel{9}x^{\frac{1}{2}}y^{42}}{w^2} \times \frac{w^{31}}{\cancel{3}x^{\frac{3}{2}}y^2}$ Converts from ÷ problem to × problem correctly.

$= 3x^{\frac{1}{2}}y^2w$

$= 3\sqrt{x}\,y^2w$ ✓✓ Correct collection of term. −1/error.

Question 3 continued.

(b) Simplify $\frac{\sqrt{5}}{\sqrt{2}}+\frac{2}{\sqrt{3}}$ (2 marks)

$= \frac{\sqrt{5}.\sqrt{3}}{\sqrt{2}.\sqrt{3}} + \frac{2.\sqrt{2}}{\sqrt{3}.\sqrt{2}}$

$= \frac{\sqrt{15} + 2\sqrt{2}}{\sqrt{6}}$ ✓ simplified denominator – not rationalised.

$= \frac{\sqrt{6}(\sqrt{15} + 2\sqrt{2})}{\sqrt{6}.\sqrt{6}}$

$= \frac{\sqrt{90} + 2\sqrt{12}}{6}$

OR $\frac{3\sqrt{10} + 4\sqrt{3}}{6}$ ✓ simplified – denominator rationalised. Accept either form.

(c) calculate the exact value of $\sin(165°)$ (4 marks)

$\sin(165°) = \sin(120° + 45°)$ ✓ concept $165° = 120° + 45°$

$= \sin(120°).\cos(45°) + \cos(120°).\cos(45°)$ ✓ correct use of rule

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2}$ ✓ correct exact values.

$= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$

or $\frac{\sqrt{6} - \sqrt{2}}{4}$ ✓ simplified Answer. ... accept either.

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Question 4 **(6 marks)**

A function is defined by $f(x) = \sqrt{3x}$.

(a) Calculate $f(12)$. (1 mark)

$f(12) = \sqrt{3 \times 12}$
$= \sqrt{36}$
$= 6$ ✓ right/wrong

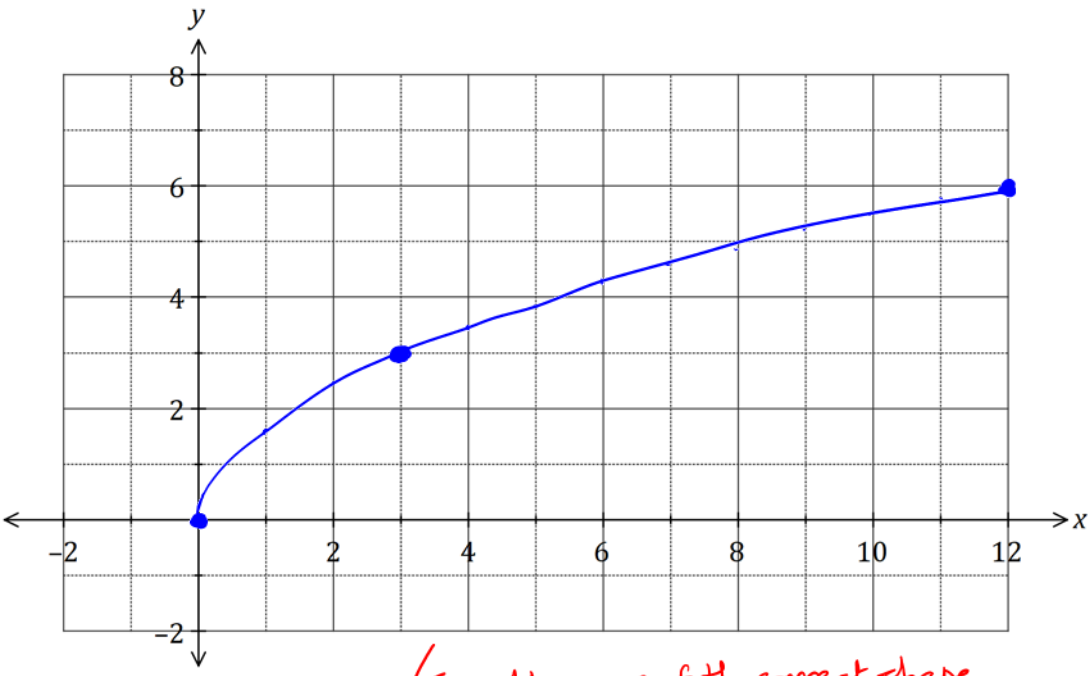
(b) State the domain and range of $f(x)$. (2 marks)

$D_f = \{x \mid x \geq 0, x \in \mathbb{R}\}$ ✓ right/wrong

$R_f = \{y \mid y \geq 0, y \in \mathbb{R}\}$ ✓ right/wrong

↑ Just this ok.

(c) Sketch the graph of $y = f(x)$ on the axes below. Indicate key points. (3 marks)



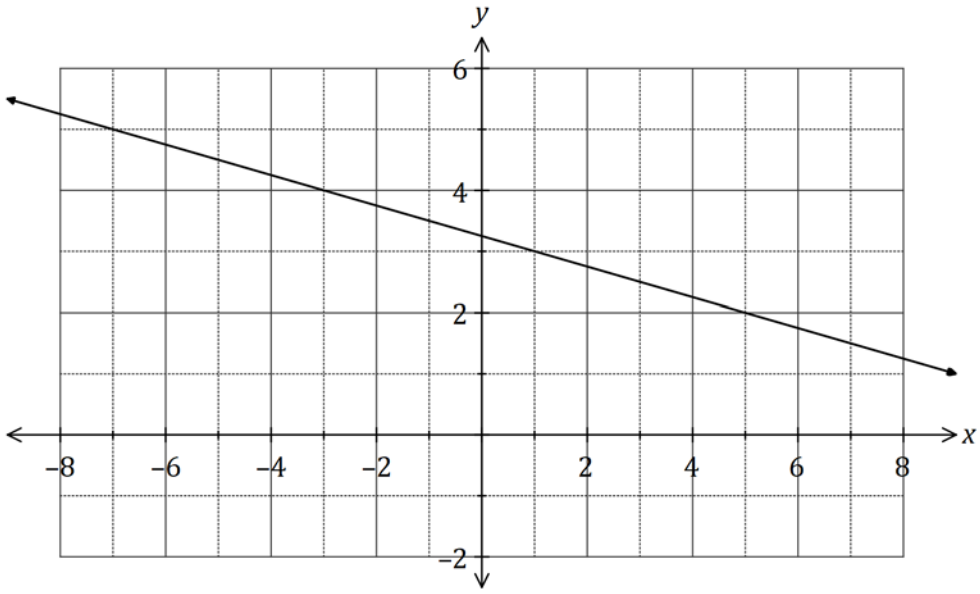
✓ Smooth curve, of the correct shape

✓ Curve passes through (3,3)

✓ Curve passes through (0,0) & (12,6)

Question 5 **(6 marks)**

The graph of the line L_1 is shown below.



(a) Determine the equation of L_1. (3 marks)

L_1 contains $(5,2)$ & $(1,3)$

$\therefore m = \frac{2-3}{5-1}$

$= -\frac{1}{4}$ ✓ correct gradient

$\therefore$ Eqn is of the form $y = -\frac{1}{4}x + c$, to solve for c, substitute $(1,3)$

$\therefore 3 = -\frac{1}{4}(1) + c$

$c = \frac{13}{4}$ ✓ correct c-value

$\therefore$ Required Equation is $y = -\frac{1}{4}x + \frac{13}{4}$ ✓ states resulting equation in a correct form

Two points are located at $A(-10, 5)$ and $B(6, 29)$.

(b) Line L_2 is perpendicular to L_1 and passes through the mid-point of A and B. Determine the equation of L_2. (3 marks)

Midpoint of AB $(-2, 17)$

$\therefore$ m of $\perp$ line is 4. ✓ correct m of $\perp$ line

$\therefore L_2$ is of the form $y = 4x + c$, to solve for c, substitute $(-2,17)$

$\therefore 17 = 4(-2) + c$

$c = 25$ ✓ correct c-value

$\therefore$ Required Equation is $y = 4x + 25$ ✓ states resulting equation in correct form

See next page

Question 6 **(6 marks)**

(a) Expand and simplify $(x+2)(2x-5)(x-2)$. (2 marks)

$= (x+2)(x-2)(2x-5)$

$= (x^2-4)(2x-5)$ ✓ correct expansion of one pair of brackets

$= 2x^3-5x^2-8x+20$ ✓ correct final expansion.

(b) One solution to the equation $x^3+36=5x^2+12x$ is $x=2$. Determine all other solutions. (4 marks)

If $x=2$ is a solution, then $x-2$ is a factor.

$$\begin{array}{r} x^2-3x-18 \\ x-2 \overline{)\ x^3-5x^2-12x+36} \\ -(x^3-2x^2) \\ \hline -3x^2-12x \\ -(-3x^2+6x) \\ \hline -18x+36 \\ -18x+36 \\ \hline 0 \end{array}$$

✓ Notion of $\div (x-2)$

correct process

$\therefore x^3-5x^2-12x+36 = (x-2)(x^2-3x-18)$

$= (x-2)(x-6)(x+3)$ ✓ completes factorisation

∴ Solutions to $x^3+36=5x^2+12x$ are $x=2$, $x=6$, $x=-3$ ✓ determines other solutions.

Can still state this one.

Question 7 **(8 marks)**

(a) Solve the equation $\sqrt{3}\tan(x) - 3 = 0$ for $0 \leq x \leq 2\pi$. (3 marks)

$\tan(x) = \frac{3}{\sqrt{3}}$

$\therefore \tan(x) = \sqrt{3}$ ✓ rearrangement

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ ✓ ✓ one for each correct solution.

(b) A function has a period of k and is defined by $f(x) = 4\cos(2x)$.

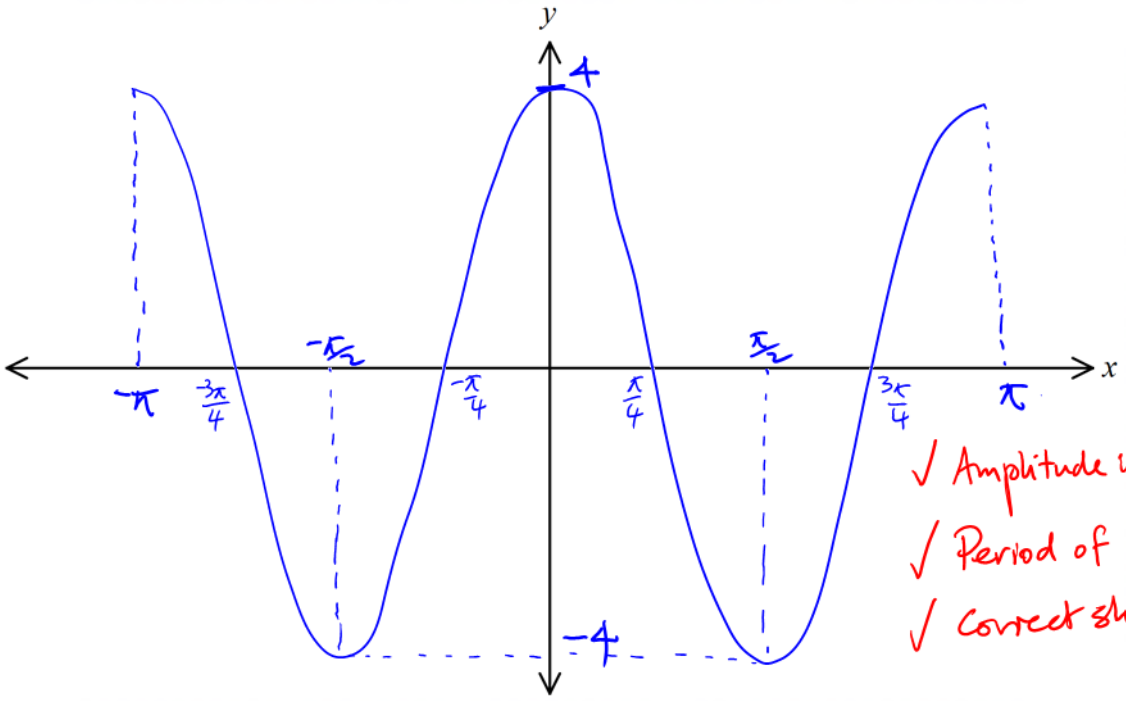
(i) State the value of k. (1 mark)

$k = \pi$ or $180°$ ✓ right/wrong.

(ii) State the amplitude of $f(x)$. (1 mark)

Amplitude = 4 right/wrong.

(iii) Sketch the graph of $y = f(x)$ over the domain $-k \leq x \leq k$. (3 marks)



✓ Amplitude is 4

✓ Period of π

✓ correct shape.

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Question 8 **(7 marks)**

(a) Calculate the value(s) of d so $h(x) = 4x^2 - dx + 3$ has two roots. (4 marks)

Require $0 < b^2 - 4ac$ ✓ uses the concept of the discriminant ie $\Delta > 0$

$\therefore 0 < d^2 - 4(4)(3)$

$\therefore 0 < d^2 - 48$ ✓ substitutes correct values

$\therefore d^2 > 48$

$\therefore d > \sqrt{48}$ or $d < -\sqrt{48}$

or $4\sqrt{3}$ ✓ Positive root

or $-4\sqrt{3}$ ✓ negative root

max of 3 out of 4 if $d = \pm\sqrt{48}$

(b) Calculate the size of the acute angle between the lines: (3 marks)

$y = x + 3$ and $y = \sqrt{3}x + 7$

Using $m = \tan\theta$

for $y = x + 3$

$m = 1$

$\therefore \tan\theta_1 = 1$

$\therefore \theta_1 = 45°$ ✓ Angle for $y = x + 3$

for $y = \sqrt{3}x + 7$

$m = \sqrt{3}$

$\tan\theta_2 = \sqrt{3}$

$\theta_2 = 60°$ ✓ for Angle for $y = \sqrt{3}x + 7$

$\therefore$ Acute angle between the lines is $60° - 45° = \underline{15°}$ ✓ correct Acute angle

Question 9 **(7 marks)**

(a) The graph of the relation $y^2 = x$ passes through the points $(16, a)$ and $(b, -5)$. Determine the values of a and b. (3 marks)

for $(16, a)$

$\therefore a^2 = 16$

$\therefore a = \pm 4$ ✓one solution of a.

✓second solution of a.

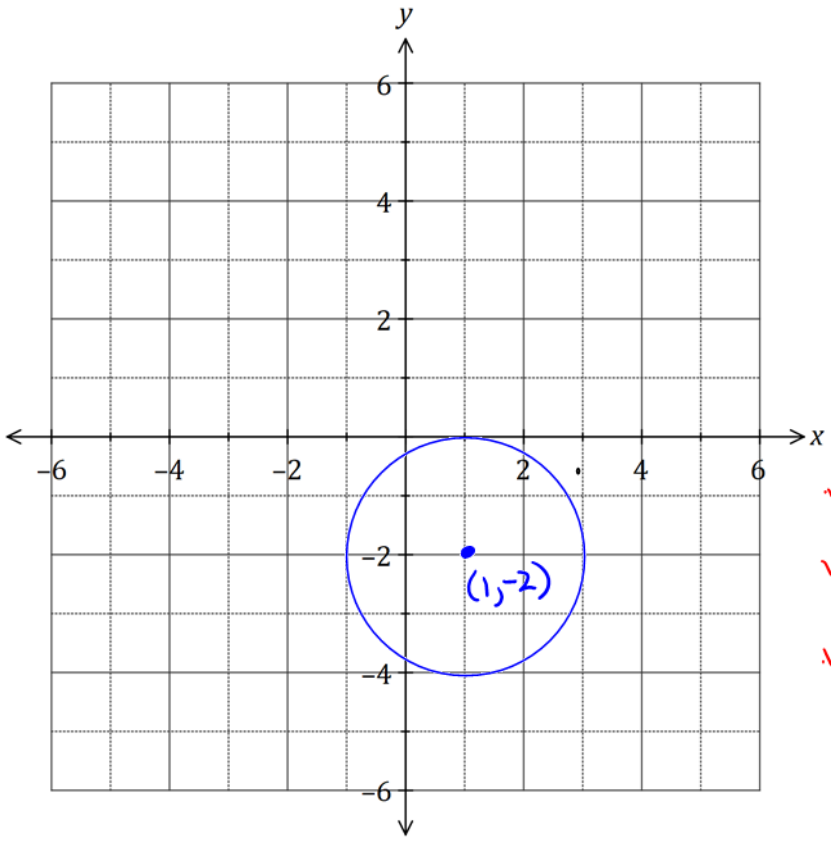
for $(b, -5)$

$(-5)^2 = b$

$\therefore b = 25$ ✓solution for b

(b) Another relation is defined by $(x - 1)^2 + (y + 2)^2 = 4$.

(i) Sketch the graph of this relation on the axes below. (3 marks)



✓ centre at (1, -2)

✓ radius 2 units

✓ Smooth curve - circle.

(ii) Provide reasoning why the graph above is of a relation rather than a function. (1 mark)

- Fails the vertical line test
- It is a one-to-many relationship
- A single x-value has two y-values

✓ Valid reason

End of Questions for Section 1

Supplementary page

Question number: _____

Supplementary page

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